United Kingdom
Mathematics Trust

Senior Mathematical Challenge

Tuesday 4 October 2022

Organised by the United Kingdom Mathematics Trust

supported by Overleaf

Candidates must be full-time students at secondary school or FE college.
England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark A, B, C, D, E on the Answer Sheet for each question. Mark only one option, boldly, within the box.
5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all markings, including bits of eraser stuck to the page, and interpret the mark in its own way.
6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. **Scoring rules:** All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.
9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until 08:00 BST on Wednesday 5 October.

Enquiries about the Senior Mathematical Challenge should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. When the expression $\dfrac{(2^2-1)\times(3^2-1)\times(4^2-1)\times(5^2-1)}{(2\times 3)\times(3\times 4)\times(4\times 5)\times(5\times 6)}$ is simplified, which of the following is obtained?

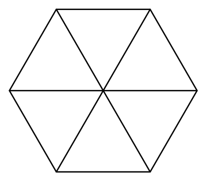
A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{1}{5}$ E $\frac{1}{6}$

2. What is the smallest prime which is the sum of five different primes?

A 39 B 41 C 43 D 47 E 53

3. The figure shows a regular hexagon.

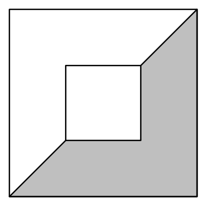
How many parallelograms are there in the figure?



A 2 B 4 C 6 D 8 E more than 8

4. The diagram shows two symmetrically placed squares with sides of length 2 and 5.

What is the ratio of the area of the small square to that of the shaded region?



A 7 : 24 B 1 : 3 C 8 : 25 D 8 : 21 E 2 : 5

5. What is the value of $\dfrac{1}{1.01}+\dfrac{1}{1.1}+\dfrac{1}{1}+\dfrac{1}{11}+\dfrac{1}{101}$?

A 2.9 B 2.99 C 3 D 3.01 E 3.1

6. What is the value of $\dfrac{4^{800}}{8^{400}}$?

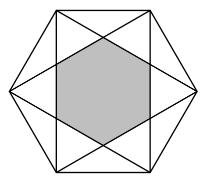
A $\frac{1}{2^{400}}$ B $\frac{1}{2^{200}}$ C 1 D 2^{200} E 2^{400}

7. In 2021, a first class postage stamp cost 85p and a second class postage stamp cost 66p. In order to spend an exact number of pounds and to buy at least one of each type, what is the smallest total number of stamps that should be purchased?

A 10 B 8 C 7 D 5 E 2

8. In the diagram, the outer hexagon is regular and has an area of 216.

What is the shaded area?



A 108 B 96 C 90 D 84 E 72

9. A light-nanosecond is the distance that a photon can travel at the speed of light in one billionth of a second. The speed of light is $3\times 10^8\,\text{ms}^{-1}$.

How far is a light-nanosecond?

A 3 cm B 30 cm C 3 m D 30 m E 300 m

10. What is the value of x in the equation $\dfrac{1+2x+3x^2}{3+2x+x^2}=3$?

A -5 B -4 C -3 D -2 E -1

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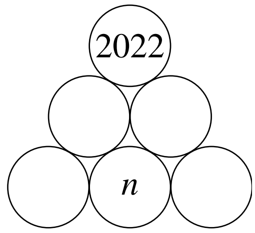
11. In the number triangle shown, each disc is to be filled with a positive integer. Each disc in the top or middle row contains the number which is the product of the two numbers immediately below.



What is the value of n?

A 1 B 2 C 3 D 6 E 33

12. What is the sum of the digits of the integer which is equal to $6666666^2 - 3333333^2$?

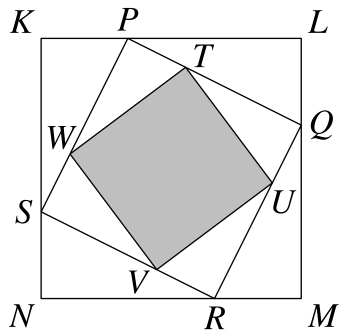
A 27 B 36 C 45 D 54 E 63

13. Three rugs have a combined area of $90\,\mathrm{m}^2$. When they are laid down to cover completely a floor of area $60\,\mathrm{m}^2$, the area which is covered by exactly two layers of rug is $12\,\mathrm{m}^2$.

What is the area of floor covered by exactly three layers of rug?

A $2\,\mathrm{m}^2$ B $6\,\mathrm{m}^2$ C $9\,\mathrm{m}^2$ D $10\,\mathrm{m}^2$ E $12\,\mathrm{m}^2$

14. The diagram shows a square, $KLMN$. A second square $PQRS$ is drawn inside it, as shown in the diagram, where P divides the side KL in the ratio $1:2$. Similarly, a third square $TUVW$ is drawn inside $PQRS$ with T dividing PQ in the ratio $1:2$.



What fraction of the area of $KLMN$ is shaded?

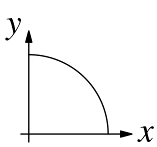
A $\dfrac{25}{81}$ B $\dfrac{16}{49}$ C $\dfrac{4}{9}$ D $\dfrac{40}{81}$ E $\dfrac{2}{3}$

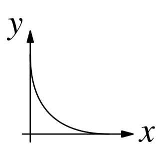
15. The hare and the tortoise had a race over 100 m, in which both maintained constant speeds. When the hare reached the finish line, it was 75 m in front of the tortoise. The hare immediately turned around and ran back towards the start line.

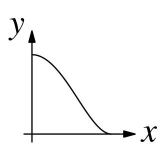
How far from the finish line did the hare and the tortoise meet?

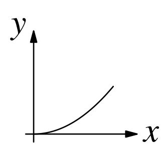
A 54 B 60 C 64 D $66\frac{2}{3}$ E 72

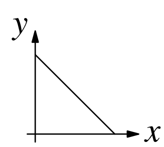
16. Which diagram could be a sketch of the curve $\sqrt{x} + \sqrt{y} = 1$?

A 

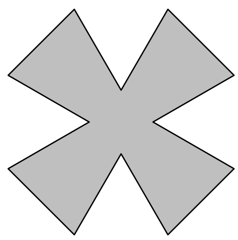
B 

C 

D 

E 

17. The shape shown is made by removing four equilateral triangles with side-length 1 from a regular octagon with side-length 1.



What is the area of the shape?

A $2 - 2\sqrt{2} + \sqrt{3}$ B $2 + 2\sqrt{2} - \sqrt{3}$ C $2 + 2\sqrt{2} + \sqrt{3}$
D $3 - 2\sqrt{2} - \sqrt{3}$ E $2 - 2\sqrt{2} - \sqrt{3}$

18. The numbers x and y are such that $3^x + 3^{y+1} = 5\sqrt{3}$ and $3^{x+1} + 3^y = 3\sqrt{3}$.

What is the value of $3^x + 3^y$?

A $\sqrt{3}$ B $2\sqrt{3}$ C $3\sqrt{3}$ D $4\sqrt{3}$ E $5\sqrt{3}$

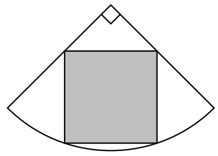
19. How many pairs of real numbers (x, y) satisfy the simultaneous equations $x^2 - y = 2022$ and $y^2 - x = 2022$?

A infinitely many B 1 C 2 D 3 E 4

20. A square is inscribed inside a quadrant of a circle. The circle has radius 10.

What is the area of the square?

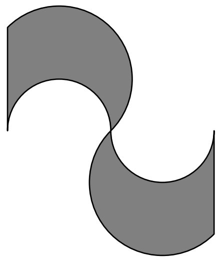
A $25\sqrt{2}$ B 36 C 12π D 40 E $30\sqrt{2}$



21. The perimeter of a logo is created from two vertical straight edges, two small semicircles with horizontal diameters and two large semicircles. Both of the straight edges and the diameters of the small semicircles have length 2. The logo has rotational symmetry as shown.

What is the shaded area?

A 4 B $4 - \pi$ C 8 D $4 + \pi$ E 12



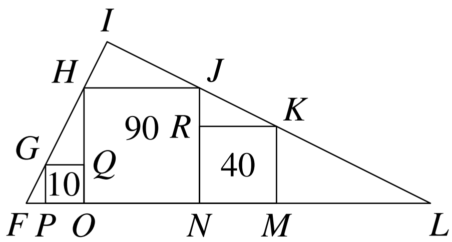
22. How many pairs of integers (x, y) satisfy the equation $\sqrt{x - \sqrt{x + 23}} = 2\sqrt{2} - y$?

A 0 B 1 C 4 D 8 E infinitely many

23. Three squares $GQOP$, $HJNO$ and $RKMN$ have vertices which sit on the sides of triangle FIL as shown. The squares have areas of 10, 90 and 40 respectively.

What is the area of triangle FIL?

A 220.5 B $\frac{21}{5}\sqrt{10}$ C 252 D $\frac{21}{2}\sqrt{10}$ E 441



24. The numbers x, y, p and q are all integers. x and y are variable and p and q are constant and positive. The four integers are related by the equation $xy = px + qy$.

When y takes its maximum possible value, which expression is equal to $y - x$?

A $pq - 1$ B $(p - 1)(q - 1)$ C $(p + 1)(q - 1)$ D $(p - 1)(q + 1)$ E $(p + 1)(q + 1)$

25. A drinks carton is formed by arranging four congruent triangles as shown. $QP = RS = 4$ cm and $PR = PS = QR = QS = 10$ cm.

What is the volume, in cm^3, of the carton?

A $\frac{16}{3}\sqrt{23}$ B $\frac{4}{3}\sqrt{2}$ C $\frac{128}{25}\sqrt{6}$ D $\frac{13}{2}\sqrt{23}$ E $\frac{8}{3}\sqrt{6}$